2021

MATHEMATICS — HONOURS

Paper : DSE-A-1

(Industrial Mathematics)

Full Marks : 65

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

1. Choose the correct answer with proper justification / explanation for each of the multiple choice question given below (For each question, one mark for each correct answer and one mark for justification) : 2×10

(a) The attenuation coefficient of an X-ray beam measures

(i) proportion of the photons absorbed by each millimeter of a substance when an X-ray passes through it.

(ii) wavelength of the X-ray.

(iii) proportion of the photons which are not absorbed by a substance when an X-ray passes through it.

(iv) None of the above.

(b) If $l_{t,\theta}$ be the line through the point $(t\cos\theta,\ t\sin\theta)$ and perpendicular to the unit vector $\hat{n} = (\cos\theta,\ \sin\theta)$, then $x + y = \sqrt{2}$ is same as

(i) $l_{1,\frac{\pi}{2}}$ (ii) $l_{1,\frac{\pi}{4}}$ (iii) $l_{0,\frac{\pi}{2}}$ (iv) $l_{\sqrt{2},\frac{\pi}{4}}$

(c) Radon transform of $f = e^{-x^2-y^2}$ is

(i) $\sqrt{\pi}e^{-p^2}$ (ii) $\sqrt{\pi}e^{p^2}$ (iii) πe^{-p^2} (iv) $\frac{\sqrt{\pi}}{2}e^{-p^2}$

(d) If f and g be defined and integrable on the real line, the convolution of f and g is defined by

(i) $(f * g)(x) = \int_{t=-\infty}^{\infty} f(t)g(x+t)\,dt$ for $x \in \mathbb{R}$

(ii) $(f * g)(x) = \int_{t=-\infty}^{\infty} f(t)g(x-t)\,dt$ for $x \in \mathbb{R}$

(iii) $(f * g)(x) = \int_{t=-\infty}^{\infty} f(t)g(xt)\,dt$ for $x \in \mathbb{R}$

(iv) $(f * g)(x) = \int_{t=-\infty}^{\infty} f(t)g(x/t)\,dt$ for $x \in \mathbb{R}$

(e) Algebraic reconstruction techniques (ARTs) are techniques for reconstructing images

(i) that have no direct connection to the Radon inversion formula.

(ii) that are same as the Radon inversion formula.

(iii) that are connected to but not same as the Radon inversion formula.

(iv) none of the above

(f) The Fourier sine transform of $\frac{x}{a^2+x^2}$, a being a constant, is given by

(i) $2\pi.e^{-ap}$ (ii) $\pi^2.e^{-ap}$

(iii) $(\pi/2).e^{-ap}$ (iv) $\pi.e^{-ap}$

(g) Let z be a complex number such that $|z| = 4$ and $arg(z) = 5\pi/6$, then $z =$

(i) $2\sqrt{3}+2i$ (ii) $-\sqrt{3}+2i$

(iii) $2\sqrt{3}-2i$ (iv) $-2\sqrt{3}+2i$

(h) The period of the function $f(x)=\sin(2x)+\frac{1}{\cos(3x)}$ is

(i) 6π (ii) 2π

(iii) π (iv) none of these

(i) The value of the integral $\int_0^\infty x^5 e^{-x^3} dx$ is

(i) 1/3 (ii) 1

(iii) 0 (iv) 2

(j) If A is a real non-singular symmetric matrix of order n, then

(i) A and A^{-1} have same set of eigenvectors.

(ii) A and A^{-1} have different set of eigenvectors.

(iii) A and A^{-1} have some common eigenvectors except one eigenvector.

(iv) none of these.

Unit – I

2. Answer ***any two*** questions :

(a) What do you mean by X-ray Computerized Tomography (CT)? Explain with example. 5

(b) (i) If z is a complex number, then find the minimum value of $|z| + |z-1|$.

(ii) For any two complex numbers z_1 and z_2 and any real numbers a and b; then show that

$$|(az_1 - bz_2)|^2 + |(bz_1 - az_2)|^2 = (a^2 + b^2)(|z_1|^2 + |z_2|^2)$$ 2+3

(c) Define (in the Hadamard sense) the well-posedness of a mathematical problem. Give an example of an ill-posed problem. 3+2

(d) Solve the following differential equation : $x\frac{d^2y}{dx^2} + (x-1)\frac{dy}{dx} - y = x^2$. 5

Unit – II

3. Answer ***any two*** questions :

(a) Direct problem is given by : a continuous function $x : [0, 1] \to R$, compute $y(t) := \int_0^t x(s)ds,\ t \in [0, 1]$.

Find its inverse problem. 5

(b) Find the inverse function of the function defined by $f(x) = -x^5$, for $x \in [-1, 1]$. Is the inverse function f^{-1} is continuous at $x = 0$? 2+3

(c) Consider the boundary value problem $\frac{d^2u}{dx^2} = f,\ u(0) = u(l) = 0$, where $f : R \to R$ is a given continuous function. Suppose that the solution u and f are known. Find the length l of the interval. 5

(d) If 5, 2, 2 are eigenvalues of a square matrix A of order 3 having eigenvectors $a\begin{pmatrix}1\\1\\1\end{pmatrix}$ and $b\begin{pmatrix}1\\0\\-1\end{pmatrix} + c\begin{pmatrix}0\\1\\-1\end{pmatrix}$ associated with 5 and 2 respectively, where $a \neq 0$, $(b, c) \neq (0, 0)$, then find the matrix A. 5

Unit – III

4. Answer ***any one*** question : 5×1

(a) An X-ray beam $A(x)$ propagates in a uniform medium which is defined by $A(x) = x$. Prove that the intensity $I(x)$ of this beam is a Gaussian distribution, with boundary conditions $\lim_{|x|\to\infty} I(x) = 0$. Find the average value of this intensity.

(b) If the intensity of an X-ray light beam is $I(x)=(2x+3)e^{-\frac{dx^2}{2}}, x>0$, then find the inhomogeneous medium and hence show that it is zero at the point $x=-\frac{3}{4}+\frac{\sqrt{16+9d}}{4\sqrt{d}}$, where d is a positive real constant.

Unit – IV

5. Answer ***any one*** question : 5×1

(a) Show that Radon transform is a linear transform.

(b) Prove that the line $\mathcal{L}_{1/2,\, \pi/6}$ has a standard form $x=\frac{\sqrt{3}}{4}-\frac{s}{2}, y=\frac{1}{4}+\frac{\sqrt{3}}{2}s$, then find the Random transformation of $f(x, y)=\begin{cases} x, & x^2+y^2 \leq 1 \\ 0, & \text{elsewhere} \end{cases}$ at the point (1/2, π/6).

Unit – V

6. Answer ***any one*** question : 5×1

(a) What are the differences between back projection and Random transformation?

(b) Find back projection of the Radon transform of a attenuation-coefficient function *f*.

Unit – VI

7. Answer ***any two*** questions : 5×2

(a) Write a short note on algebraic reconstruction technique on the base of CT scan.

(b) If $f(x)$ is an absolutely integrable and piecewise continuous function with a point of discontinuity at $x = \alpha$ but $\lim\limits_{x\to\alpha^-} f(x)$ and $\lim\limits_{x\to\alpha^+} f(x)$ exist, then prove that

$$\mathcal{F}^{-1}(\mathcal{F} f)(\alpha)=\frac{1}{2}\left(\lim_{x\to\alpha^-} f(x)+\lim_{x\to\alpha^+} f(x)\right).$$

(c) Show that the inverse Fourier transform of an even function is a real-valued function and the inverse Fourier transform of an odd function is a purely imaginary function.

(d) Find the inverse Fourier transform of the function $F(w)=\frac{2}{1+w^2}$.